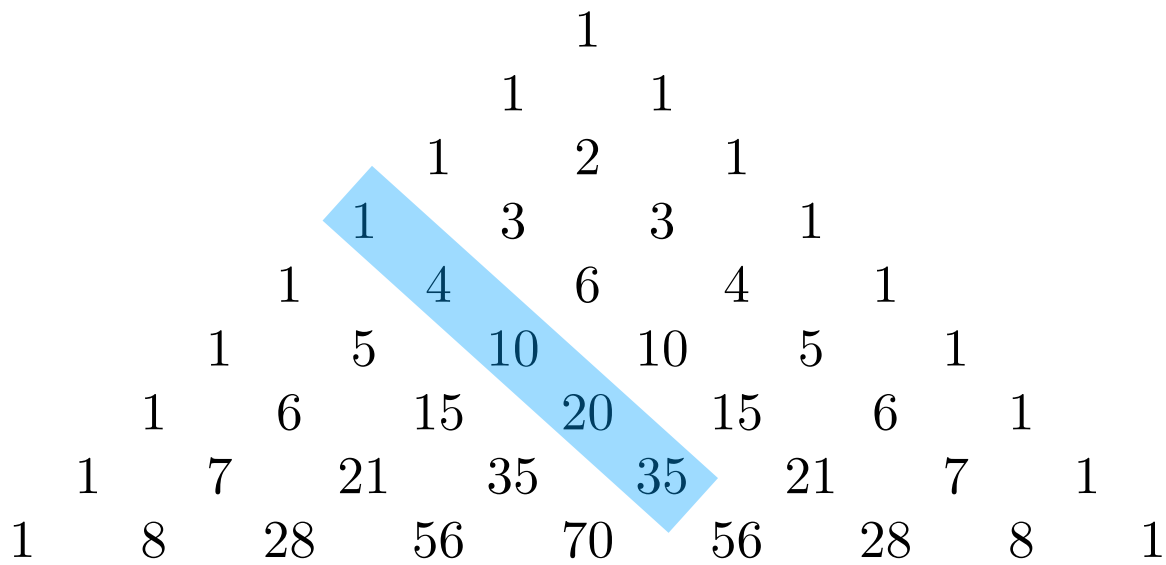
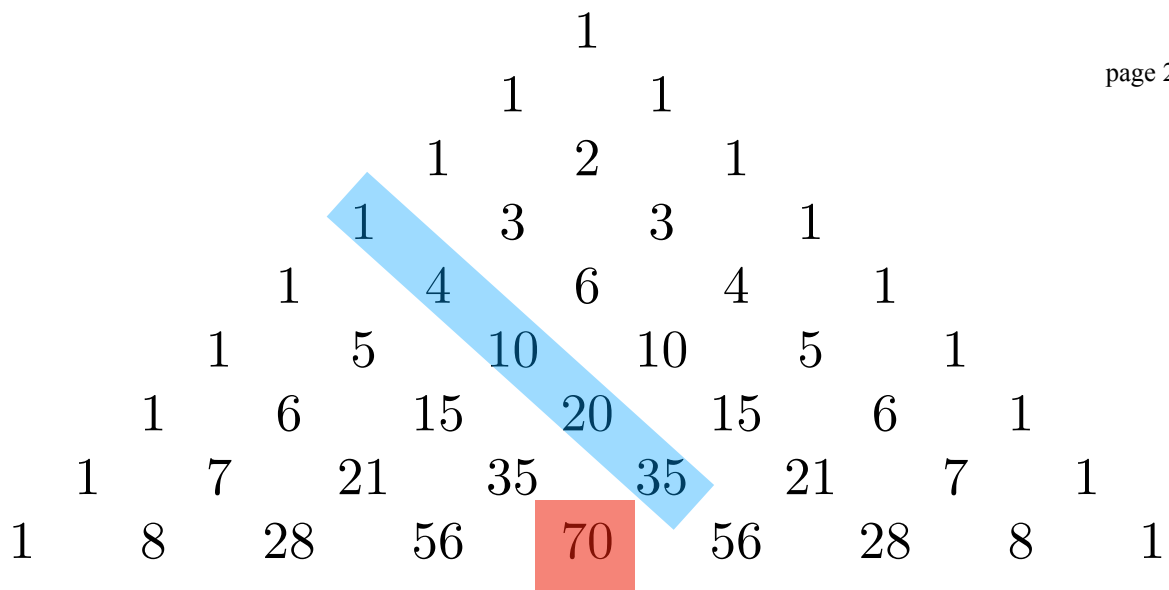
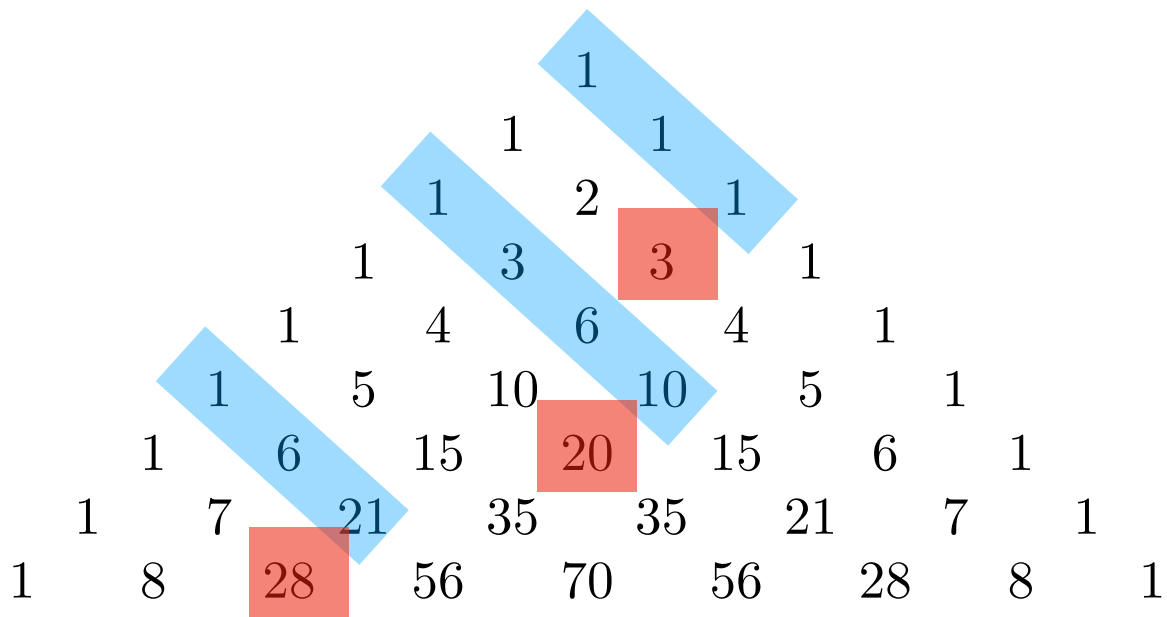


CISC-102
Winter 2020
Week 9 Addendum



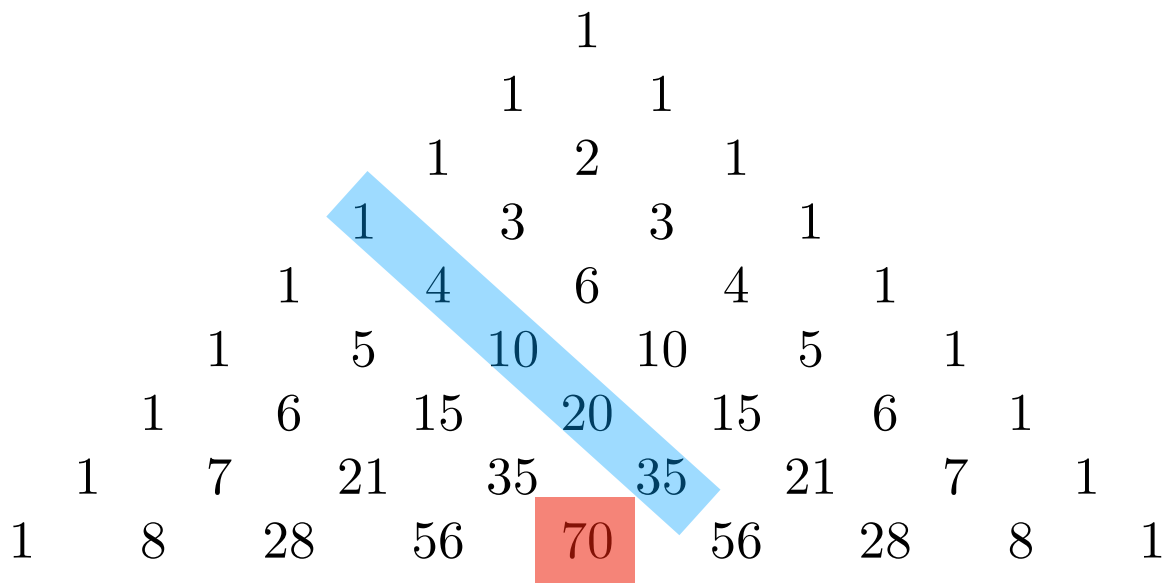
More Fun with Pascal's Triangle.





$$\binom{n}{0} + \binom{n+1}{1} + \dots + \binom{n+k}{k} = \sum_{i=0}^k \binom{n+i}{i} = \binom{n+k+1}{k-1}$$

We can prove this result using mathematical induction.



Theorem: Let m be any natural number, then

$$\binom{m}{0} + \binom{m+1}{1} + \cdots + \binom{m+n}{n} = \sum_{i=0}^n \binom{m+i}{i} = \binom{m+n+1}{n}$$

for all Natural numbers n .

Proof: We prove the result using induction on n .

Base: For $n = 1$,

$$\binom{m}{0} + \binom{m+1}{1} = m + 2 = \binom{m+2}{1}$$

Induction hypothesis: Assume:

$$\binom{m}{0} + \binom{m+1}{1} + \dots + \binom{m+k}{k} = \sum_{i=0}^k \binom{m+i}{i} = \binom{m+k+1}{k}$$

for a fixed natural number $k \geq 1$.

Induction Step: We want to show that the induction hypothesis implies:

$$\binom{m}{0} + \binom{m+1}{1} + \dots + \binom{m+k}{k} + \binom{m+k+1}{k+1} = \sum_{i=0}^{k+1} \binom{m+i}{i} = \binom{m+k+2}{k+1}$$

Thus we have:

$$\begin{aligned} \binom{m}{0} + \binom{m+1}{1} + \dots + \binom{m+k}{k} + \binom{m+k+1}{k+1} &= \binom{m+k+1}{k} + \binom{m+k+1}{k+1} \\ &= \binom{m+k+2}{k+1} \end{aligned}$$

					1						
					1	1					
				1	2	1					
			1	3	3	1					
			1	4	6	4	1				
			1	5	10	10	5	1			
			1	6	15	20	15	6	1		
			1	7	21	35	35	21	7	1	
			1	8	28	56	70	56	28	8	1

The Hexagon Identity:

$$\binom{n-1}{k-1} \binom{n}{k+1} \binom{n+1}{k} = \binom{n-1}{k} \binom{n}{k-1} \binom{n+1}{k+1}$$

Algebraic Proof:

$$\binom{n-1}{k-1} \binom{n}{k+1} \binom{n+1}{k} = \frac{(n-1)!}{(k-1)!(n-k)!} \times \frac{(n)!}{(k+1)!(n-k-1)!} \times \frac{(n+1)!}{(k)!(n-k+1)!}$$

$$\binom{n-1}{k} \binom{n}{k-1} \binom{n+1}{k+1} = \frac{(n-1)!}{(k)!(n-k-1)!} \times \frac{(n)!}{(k-1)!(n-k+1)!} \times \frac{(n+1)!}{(k+1)!(n-k)!}$$